SEM-2 (MATHEMATICS GENERAL)

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BMG2CC1B (Differential Equations)

Rules to Find an Integrating Factor of the differential Equation $Mdx + Ndy = 0$.

1) Rule-1 (By Inspection): Often an I.F. of given equation $Mdx + Ndy = 0$ can be found out by inspection.

Ex:1 Solve $ydx - xdy + (1+x^2)dx = 0$

Ans: Dividing each term of the given equation by x^2 (I.F.) we get

$$\frac{ydx - xdy}{x^2} + \frac{1+x^2}{x^2}dx = 0$$

$$\text{or } -\frac{xdy - ydx}{x^2} + \left(\frac{1}{x^2} + 1\right)dx = 0$$

$$\text{or } -d\left(\frac{y}{x}\right) + \left(1 + \frac{1}{x^2}\right)dx = 0$$

Integrating $-\frac{y}{x} + x - \frac{1}{x} = c$, where c is an arbitrary constant

Ex:2 Solve $y(2xy + e^x)dx = e^y dy$

Ans: Re-writing the given equations we get

$$2xy^2dx + ye^x dx - e^y dy = 0 \quad \cdots (1)$$

Dividing each term of (1) by $(I.F.)y^2$ we get

$$2xdx + \frac{ye^x dx - e^y dy}{y^2} = 0$$

$$\text{or } 2xdx + d\left(\frac{e^x}{y}\right) = 0$$

Integrating we have $x^2 + \frac{e^x}{y} = c$ where c is an arbitrary constant.

Ex:3 Solve: $(1+xy)ydx + x(1-xy)dy = 0$

Ans: Re-writing, the given equation is

$$y\,dx + x\,dy + xy(y\,dx - x\,dy) = 0 \quad \text{---(1)}$$

Dividing each term of (1) by x^2y^2 (I.F.) we

$$\frac{d(xy)}{(xy)^2} + \frac{dx}{x} - \frac{dy}{y} = 0$$

Integrating we get,

$$-\frac{1}{xy} + \log\left(\frac{x}{y}\right) = c$$

Home Work

Solve;

1) $x\,dy - y\,dx = (x^2 + y^2)\,dx$

2) $x \cdot \frac{dy}{dx} + y = y^2 \log x$

3) $x\,dx + y\,dy + a\sqrt{x^2+y^2}\,dx = 0$

4) $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2+y^2} = 0$

5) $e^x dx + (xe^y + 2y)\,dx = 0$